

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
Level 3 GCE**

Centre Number

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Candidate Number

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Afternoon (Time: 2 hours)

Paper Reference **9MA0/02**

Mathematics

Advanced

Paper 2: Pure Mathematics 2

You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical formulae
stored in them.**

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1 The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}} \, dx$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} \, dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} \, dx = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)



Question 1 continued

(Total for Question 1 is 5 marks)



2. Relative to a fixed origin, points P , Q and R have position vectors \mathbf{p} , \mathbf{q} and \mathbf{r} respectively.

Given that

- P , Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p}) \quad (3)$$

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Question 2 continued

(Total for Question 2 is 3 marks)



3. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x) = \log(x+8)$$

giving a reason for your answer.

(2)



Question 3 continued

(Total for Question 3 is 5 marks)



4. In the binomial expansion of

$$(a + 2x)^7 \quad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a .

(3)

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Question 4 continued

(Total for Question 4 is 3 marks)



5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P .
Find, using algebra, the exact x coordinate of P . (4)



Question 5 continued

(Total for Question 5 is 4 marks)



6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \quad x \in \mathbb{R} \quad x \neq -2$$

find the values of the constants A , B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} \, dx$$

giving your answer in the form $a + b \ln 2$ where a and b are integers to be found.

(4)



Question 6 continued



Question 6 continued

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Question 6 continued

(Total for Question 6 is 7 marks)



7.

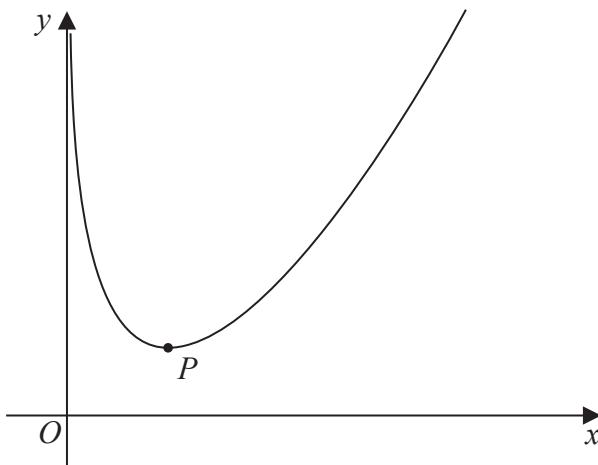
**Figure 1**

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4 \ln x \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}} \quad (4)$$

The point P , shown in Figure 1, is the minimum turning point on C .

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}} \quad (3)$$

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)





Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)



8. A curve C has equation $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $f(x)$

find, in simplest form, $f(x)$

(6)





Question 8 continued

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Question 8 continued

(Total for Question 8 is 6 marks)



P 6 6 7 8 6 A 0 2 3 5 2

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, $\theta^\circ\text{C}$, at time t seconds after heating began, is modelled by the equation

$$\theta = A - B\text{e}^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18°C
- after 10 seconds the temperature of the ethanol was 44°C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78°C

(b) Use this information to evaluate the model.

(2)



Question 9 continued

(Total for Question 9 is 6 marks)



10.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4 \cos^3 A - 3 \cos A \quad (4)$$

(b) Hence solve, for $-90^\circ \leq x \leq 180^\circ$, the equation

$$1 - \cos 3x = \sin^2 x \quad (4)$$





Question 10 continued

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Question 10 continued

(Total for Question 10 is 8 marks)



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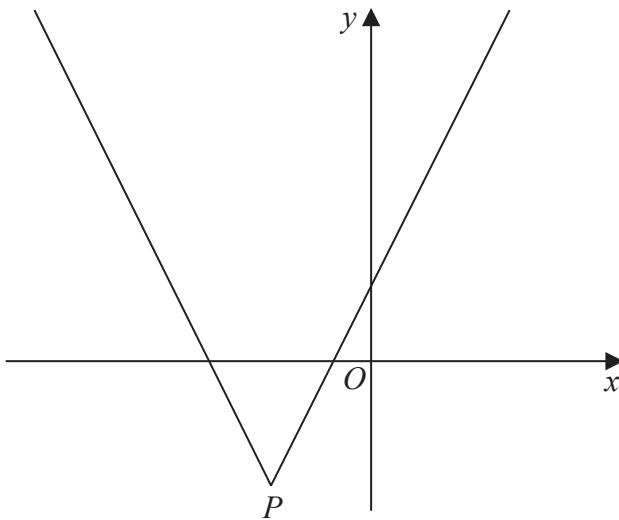


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P , shown in Figure 2.

(a) Find the coordinates of P .

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a , writing your answer in set notation.

(3)



Question 11 continued



Question 11 continued

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Question 11 continued

(Total for Question 11 is 7 marks)



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12.

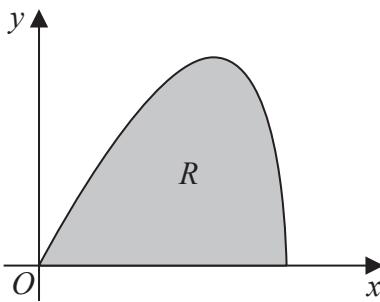


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6 \sin t \quad y = 5 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

(a) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$ (3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20 (3)

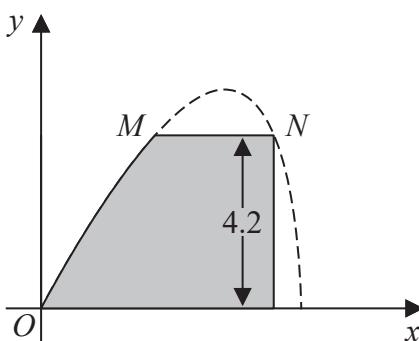


Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4. Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway. (5)



Question 12 continued



Question 12 continued

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Question 12 continued

(Total for Question 12 is 11 marks)



13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \quad x > 0 \quad x \neq k$$

where k is a constant.

(a) Deduce the value of k .

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g .

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)





Question 13 continued

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Question 13 continued

(Total for Question 13 is 6 marks)



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14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x -axis and touches the y -axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C ,

(b) find the two possible values of r , giving your answers as fully simplified surds.

(4)



Question 14 continued



Question 14 continued

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Question 14 continued

(Total for Question 14 is 7 marks)



P 6 6 7 8 6 A 0 4 5 5 2

15.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a .

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r} \quad (4)$$

Given also that S_{10} is four times S_5

(b) find the exact value of r .

(4)





Question 15 continued

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Question 15 continued

(Total for Question 15 is 8 marks)



16. Use algebra to prove that the square of any natural number is either a multiple of 3 or one more than a multiple of 3

(4)

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(Total for Question 16 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

